Easily Find Wetted Surface Area of Side Heads. Simple formulas fill void in calculations for three common types.

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Easily Find the

Wetted Surface Area of Side Heads

Simple formulas fill void in calculations for three common types | By Leonid Korelstein, Piping Systems Research and Engineering Co.

A TYPICAL problem faced by process and safety engineers is calculating the wetted area of tanks and vessels — for example, when determining the required relief load for a fire case. This area is the sum of the wetted areas of the unit's parts, which may consist of shells and heads of different types. Of course, numerical integration can provide the answer. However, having exact or accurate approximate analytical formulas allows an engineer to do this calculation quickly and simply by hand or in a spreadsheet.

Several authors [1–5] have investigated and reviewed this subject; appropriate formulas are known for many types of shells/heads. However, three types of heads — side elliptical, dished and torispherical — have lacked proper equations. Unfortunately, these types of heads are used very often.

SIDE ELLIPICAL HEAD

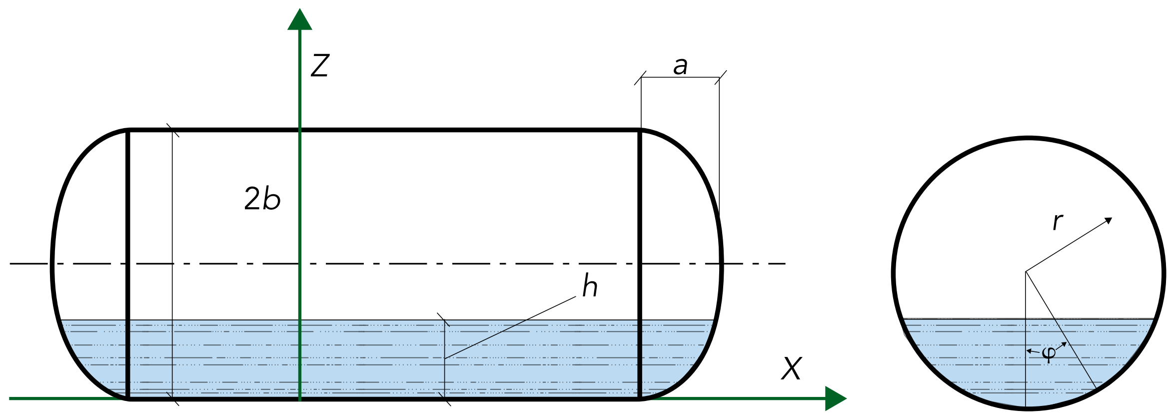

Figure 1. Drawing shows key parameters of vessel with such heads.

APPROXIMATION APPROACH

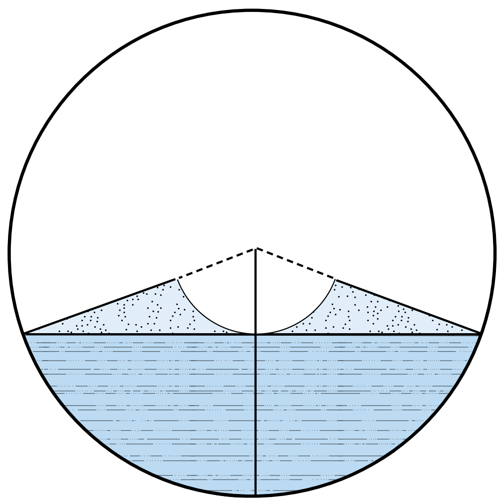

Figure 2. Method finds the difference in surface area between the spheroidal strip and the area of two triangles.

This article solves that problem, proposing general exact or accurate approximate equations for wetted area for those three types of side heads.

SIDE ELLIPTICAL HEAD

Let us first consider this configuration (Figure 1) — with semi-axis lengths a and b, $a < b$. We need to find head wetted area $A_w(h)$ depending on liquid level height h.

The eccentricity e of the elliptical head is defined as $e = (1 - a^2/b^2)^{½} < 1$.

Doane [6] proposed an approximate formula that became popular and widely used. However, the Doane equation:

- appeared "from nowhere" with no explanation of its derivation;
- was intended only for use in the specific case $a/b = 0.5$ ($e = (3)^{½}/2 \approx 0.866$); and
- was not very accurate. (Jones [4] demonstrated the equation significantly under-predicts real values of wetted area for $b < h < 2b$ and strongly over-predicts them for $0 < h < b$.)

It is possible to derive a more-general and more-accurate (as well as relatively simple and easy-to-use) formula. A good approximation of it results from the following trick (Figure 2): calculating the area $A_w(h)$ as the difference in surface area between the spheroidal strip $b - h \le r \le b$, $|\varphi| \le \arccos(1 - h/b)$ and area of the two triangles (defined by $b - h \le r \le (b - h)/\cos\varphi$, $|\varphi| \le \arccos(1 - h/b)$). You can obtain the strip area analytically and accurately approximate the triangles' area (by replacing the spheroidal surface with a conical one to allow analytical integration).

The resulting equations can be written in the form:

$$A_w(h) \approx b^2\{\cos^{-1}(X)[Y^{1} + ((1 - e^2)/e)\tanh^{-1}(eY)] - X(1 - e^2X^2)^{½}\} \text{ for } 0 < h < b \quad (1a)$$

$$A_w(h) = A_w(2b) - A_w(2b - h) \text{ for } b < h < 2b \quad (1b)$$

where $A_w(2b) = 2A_w(b) = \pi b^2[1 + ((1 - e^2)/e)\tanh^{-1}(e)]$, $H = h/b$, $X = 1 - H$ and $Y = (1 - X^2)^{½}/(1 - e^2X^2)^{½}$.

Figure 3 compares the values of (dimensionless) wetted area (for the case $e = (3)^{½}/2 \approx 0.866$) calculated by the Doane formula, the equations above, and the exact value (from numerical calculation of the area).

As Figure 3 shows, the results from Equations 1a and 1b practically coincide with those obtained by the numerical method while the Doane formula gives significant error.

In fact, Equation 1 provides accuracy within 1% for $0.5 \le F = h/2b \le 1$ and ≈3% or better for $0.1 \le F \le 0.5$. Over the interval $0 \le F \le 0.1$, the accuracy of Equations 1a and 1b drops to ≈6%. However, for very small liquid levels ($F < 0.03$), the wetted area of the correspondent "semi-cap" of spheroid can be more accurately calculated by a simple equation:

$$A_w(h) \approx \pi a h \quad (2)$$

Equation 2 transforms the known equation for the area of a spherical cap ($A = 2\pi Rh$) to the general case of a "small

COMPARISON OF RESULTS

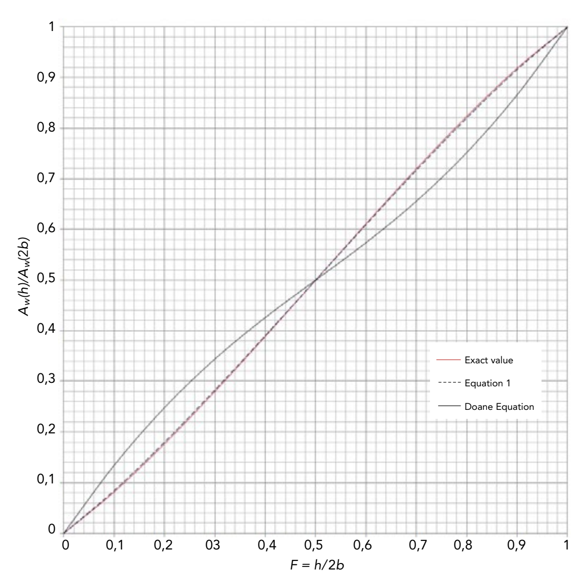

Figure 3. Calculations using Equation 1 closely match exact values while Doane equation gives significant errors.

cap" of convex surface with positive curvature: $A \approx 2\pi h/K^{½}$ where K is the Gaussian curvature of surface. In the bottom point of the head, main curvature radiuses are b and a^2/b, so $K = a^{-2}$, which leads to Equation 2.

SIDE DISHED HEAD

Let us now consider the case of side dished head, i.e., a head with radius R and height a that is part of a spherical surface of radius R_d. In this case, it is possible to derive an exact analytical equation for wetted surface area using spherical trigonometry.

The resulting formula, which is valid for all liquid levels ($0 \le h \le 2R$), can be written as:

$$A_w(h) = (R^2 + a^2)\{(\pi/2) - ((1 + E^2)/2E^2)\sin^{-1}(\Delta_1\Delta_2) + ((1 - E^2)/2E^2)\sin^{-1}\Delta_1 - ((1 - H)/E)\cos^{-1}(\Delta_2)\} \quad (3)$$

where $\Delta_1 = 1 - H$, $\Delta_2 = (1 - E^2)/((1 + E^2)^2 - 4E^2(\Delta_1)^2)^{½}$, $H = h/R$ and $E = a/R$.

SIDE TORISPHERICAL HEAD

Such a head (of diameter D and radius R) is the most complex type of head considered in this article. It (Figure 4) consists of a dished head (with radius of sphere $R_f = fD = 2fR$, $f > 0.5$) and a transitional part of torus with radiuses $R_k = kD = 2kR$ and $R - R_k$. The parameters of the head are related to each other by the following equations [1] to provide a smooth connection:

$$\sin\alpha = (1 - 2k)/2(f - k),\ R_1 = 0.5D_1 = 2fR\sin\alpha = 2fR(1 - 2k)/2(f - k),\ a_1 = R_f(1 - \cos\alpha) \text{ and } a_2 = R_k\cos\alpha \quad (4)$$

You can calculate the wetted area of the head $A_w(h)$ as the sum of the wetted area $A_{wf}(h)$ of the dished part and the wetted area $A_{wk}(h)$ of the toric part: $A_w(h) = A_{wf}(h) + A_{wk}(h)$.

When $|R - h| \le R_1$, you can calculate $A_{wf}(h)$ with Equation 3 using R_1 and a_1 instead of R and a, and $H = (h - R + R_1)/R_1$. For $h < R - R_1$, $A_{wf}(h) = 0$, and for $h > R + R_1$, $A_{wf}(h) = \pi(R_1^2 + a_1^2)$.

For $A_{wk}(h)$, you can use the same approach as for an elliptical head:

$$A_{wk}(h) = A_{wk}(2R) - A_{wk}(2R - h) \text{ for } h \ge R \quad (5)$$

with $A_{wk}(2R) = 2A_{wk}(R) = 2\pi R_k[a_2 + (R - R_k)((\pi/2) - \alpha)]$, or

$$A_{wk}(h) \approx \pi h(R_k R/X)^{½} \text{ for } 0 < h \le R - R_1 \quad (6)$$

where $X = 1 - h/R$.

This is the same $A_{wk}(h) \approx \pi h/K^{½}$ equation as for "cap" but with an additional empirical coefficient $(1/X)^{½}$ that increases accuracy. Equation 6 allows predicting $A_w(h)$ within 3% accuracy for typical f and k values.

For $R - R_1 < h < R$, applying the same approach as for an elliptical head results in the following approximation:

$$A_{wh}(h) \approx 2R_k[a_2 + (R - R_k)((\pi/2) - \alpha)]\cos^{-1}(X) + (2R_k R_1/a_2)\{R_1[\cos^{-1}(X) - \cos^{-1}(X_1)] - RX[\cosh^{-1}(X^{-1}) - \cosh^{-1}(X_1^{-1})]\} \quad (7)$$

where $X = 1 - h/R$ and $X_1 = XR/R_1$.

Equation 7 provides accuracy within 1% except for h values close to $R - R_1$, where accuracy drops to 6%. In such a case, another approximation provides accuracy of about 3%:

$$A_{wk}(h) \approx \pi h(R_k R/X)^{½} - (2R_k R_1/a_2)[R_1\cos^{-1}(X_1) - RX\cosh^{-1}(X_1^{-1})] \quad (8)$$

Equation 8 was derived from Equation 6 and an approximation of the "missing area" of the "cap."

SIDE TORISPHERICAL HEAD

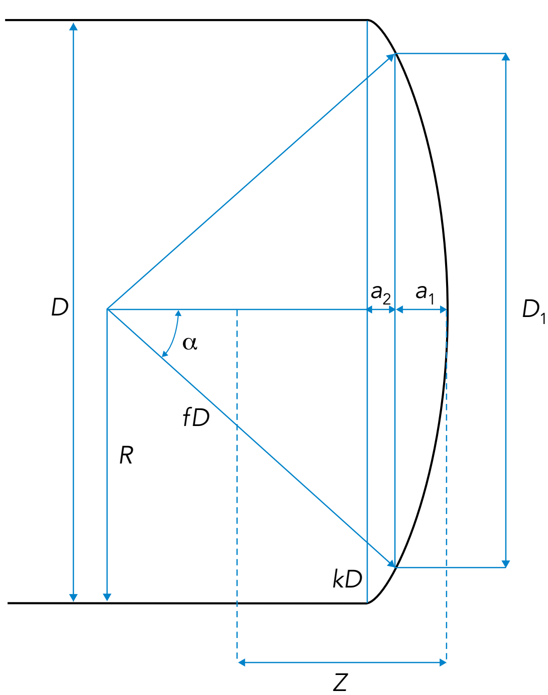

Figure 4. This relatively complex type of head requires its own equations.

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